

Test Booklet Serial No. :
OMR Sheet No. :
Roll No.


## Name \& Signature of Invigilator/s

Signature:
$\qquad$

Signature: $\qquad$
Name : $\square$

Time : 1 Hour 15 Minutes
Maximum Marks : 100

## Number of Questions in this Booklet : 50

## Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
(i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
(ii) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
Example : (A) B (D)
where (C) is the correct response.
5. Your responses to the questions are to be indicated in the OMR Sheet kept inside the Paper I Booklet only. If you mark at any place other than in the ovals in the Answer Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet soon after the examination.
11. Use only Blue/Black Ball point pen.
12. Use of any calculator or log table etc., is prohibited.
13. There is no negative marks for incorrect answers.

## MATHEMATICAL SCIENCE <br> Paper - II

Note: This paper contains (50) fifty objective type questions, each question carrying two (2) marks. Attempt all the questions.

1. Which one of the following is true ?
(A) $\pi^{3}=3^{\pi}$
(B) $\pi^{3}<3^{\pi}$
(C) $3^{\pi}<\pi^{3}$
(D) $\pi^{3} \geq 3^{\pi}$
2. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^{n+1}}{n!} z^{n}$ is
(A) $\infty$
(B) 0
(C) 1
(D) $\frac{1}{2}$
3. Which one of the following statements is false?
(A) A countable union of countable sets is countable
(B) The set of all sequences whose elements are the digits 0 and 1 is countable
(C) The set of all real numbers $x$ such that $0 \leq x \leq 1$ is uncountable
(D) The set $\square^{+} \times \square^{+}$is countable
4. Which one of the following statements is false?
(A) Every convergent sequence is bounded
(B) A monotonic sequence is convergent if and only if it is bounded
(C) Every bounded sequence has a convergent subsequence
(D) Every bounded sequence is convergent
5. Let $\mathbb{F}_{5}[x]$ denote the polynomial ring over the field $\mathbb{F}_{5}$ with 5 elements. Which one of the following statements is true?
(A) $\mathbb{F}_{5}[x]$ is not a principal ideal domain
(B) $\mathbb{F}_{5}[x]$ has infinitely many quotient rings whose cardinality is bigger than a given number
(C) Prime ideals of $\mathbb{F}_{5}[x]$ are all maximal
(D) $\mathbb{F}_{5}[x]$ is not an integral domain
6. Let p be a prime number and G be a group of order $p^{4}$. Then
(A) G always has an element of order exactly $p^{4}$
(B) G always has an element of order exactly $p^{3}$
(C) G always has an element of order exactly $p^{2}$
(D) G always has an element of order exactly $p$
7. Let $S=\{x \mid x$ is an integer divisible by 20\}. Then
(A) $S+1=\{x+1 \mid x \in S\}$ is closed under addition
(B) $S+11=\{x+11 \mid x \in S\}$ is closed under addition
(C) S is closed under multiplication
(D) $S$ is not closed under addition
8. Let $S=\{1,2,3,4,5,6\}$. Then the cardinality of the set of permutations of $S$ which fixes exactly two elements is
(A) 24
(B) 36
(C) 135
(D) 120
9. The sum of the series $1+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{4}+\ldots$ is
(A) $\log 2 \sqrt{3}$
(B) $\log 2 \sqrt{2}$
(C) 0
(D) $\log \sqrt{6}$
10. Which one of the following statements is false?
(A) Any closed subspace of a compact space is compact
(B) Any compact subset of a Hausdorff space is closed
(C) A subset of a Euclidean space is compact if and only if it is closed and bounded
(D) The set $\left\{\left.\frac{1}{n} \right\rvert\, n \in \square^{+}\right\}$is compact in $\mathbb{R}$
11. Which one of the following statements is true?
(A) A second countable space is first countable
(B) The product of two Lindelöf spaces is Lindelöf
(C) A subspace of a separable space is separable
(D) $\mathbb{R}$, with lower limit topology, is metrizable
12. The boundary of $A=\{x \times 0 \mid 0 \leq x<1\}$ in $\mathbb{R}^{2}$ with usual topology is
(A) $\{0,1\}$
(B) $\{0 \times 0,0 \times 1\}$
(C) $\{x \times 0 \mid 0 \leq x \leq 1\}$
(D) $\{x \times 0 \mid 0 \leq x<1\}$
13. Let $\mathrm{T}: \mathrm{V} \rightarrow \mathrm{V}$ be a linear transformation of a vector space $V$ over $\mathbb{R}$, with characteristic polynomial $x^{4}+2 x^{2}+1$. How many linearly independent eigen-vectors does T have ?
(A) 0
(B) 1
(C) 2
(D) 4
14. Let $x_{0}$ be any fixed real number. Let $\left\{x_{n}\right\}$ be a sequence of real numbers defined inductively by the relation $x_{n}=\frac{1+x_{n-1}}{2}$. Then the sequence $\left\{x_{n}\right\}$
(A) Converges to 1
(B) Converges to $x_{0}$
(C) Diverges to $\infty$
(D) Does not have a limit
15. Let W be a subspace of a real vector space $V$ with basis $\left\{w_{1}, w_{2}, w_{3}, w_{4}\right\}$ and $U$ be a subspace of $V$ with basis $\left\{w_{1}, w_{2}, w_{3}, u_{1}, u_{2}\right\}$. Then dim $(U \cap W)$ has to be
(A) $\geq 3$
(B) $\leq 3$
(C) 3
(D) 4
16. Let V be a finite dimensional vector space over a field $F$, $S$ be a nonempty subset of V . Which one of the following statements is true?
(A) S always contains a basis of V if $\operatorname{dim}_{F} V$ is finite
(B) If $S$ is infinite and $\operatorname{dim}_{F} V$ is finite, then $S$ always contains a basis of $V$
(C) If $S$ is finite of cardinality $>1$ and $\operatorname{dim}_{F} V$ is infinite, then $S$ cannot contain an element which is a part of a basis of V .
(D) If $S$ is finite and cardinality of $S$ is greater than one then $S$ contains a basis of a subspace of $V$

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17. Determinant of the matrix
$\left(\begin{array}{ccc}3 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11\end{array}\right)$ is
(A) Zero over the field $\mathbb{F}_{2}$ with two elements
(B) Zero over the field
(C) Nonzero over the field $\mathbb{F}_{2}$ with two elements
(D) Nonzero over the field $\mathbb{F}_{3}$ with two elements
18. Let $\mathrm{T}: \mathrm{V} \rightarrow \mathrm{W}$ be a linear mapping where V and W are vector spaces over $\mathbb{R}$ of dimensions 5 and 6 respectively. Which one of the following need not be true?
(A) Nullity of $\mathrm{T} \leq 5$
(B) Rank of $\mathrm{T} \leq 6$
(C) Rank of $\mathrm{T} \leq 5$
(D) Rank of $\mathrm{T} \leq$ Nullity of T
19. Let $\left\{x_{n}\right\}$ be a sequence defined by $x_{1}=1, x_{n+1}=\frac{3+2 x_{n}}{2+x_{n}}, n \geq 1$ then $\left\{x_{n}\right\}$
(A) Converges to 2
(B) Converges to $\frac{3}{2}$
(C) Converges to $\sqrt{3}$
(D) Diverges
20. Let $a, b, c, d$ be real numbers with $a d-b c=1$. Define the meromorphic function $f$ by $f(z)=\frac{a z+b}{c z+d}$. Then $f$ maps the
(A) Upper half plane to itself
(B) Upper half plane to lower half plane
(C) Lower half plane to right half plane
(D) Upper half plane to right half plane
21. Suppose $f(z)$ is analytic in the entire complex plane and bounded. Then $f(z)$ must be
(A) $\sin z$
(B) $\cos z$
(C) a polynomial of degree greater than one
(D) a constant
22. The locus of the curve given by $|z+2|=|z-2|, z \in \mathbb{C}$
(A) is an ellipse
(B) is a circle
(C) consists of only the origin o
(D) is the $y$-axis
23. Suppose $f(z)=u(x, y)+i v(x, y)$ is a nonconstant analytic function on $\mathbb{C}$. Consider the families of curves defined by $u(x, y)=$ constant and $v(x, y)=$ constant. Then these families
(A) never intersect
(B) are always parallel
(C) are asymptotic to the line $y=x$
(D) are always orthogonal
24. Let $f(x, y)=y^{2}+4 x y+3 x^{2}+x^{3}$. Then
(A) $(0,0)$ is a minimum point of $f$
(B) $(0,0)$ is a saddle point of $f$
(C) $(0,0)$ is a maximum point of $f$
(D) $\left(\frac{2}{3},-\frac{4}{3}\right)$ is a maximum point of $f$
25. Let $\mathrm{f}, \mathrm{g}:[0,1] \rightarrow[0,+\infty)$ be continuous functions satisfying
$\sup f(x)=\sup g(x)$
$0 \leq x \leq 1 \quad 0 \leq x \leq 1$
Then
(A) $f(t)<g(t)$ for all $t \in[0,1]$
(B) $f(t)>g(t)$ for all $t \in[0,1]$
(C) $f(t)=g(t)$ for some $t \in[0,1]$
(D) $f(t) \neq g(t)$ for all $t \in[0,1]$
26. The third approximation to the solution of the differential equation $\frac{d y}{d x}=1+x y$, $y(0)=1$ in Picard's method is
(A) $1+x+\frac{x^{2}}{3}+\frac{x^{3}}{2}+\frac{x^{4}}{8}+\frac{x^{5}}{15}+\frac{x^{6}}{48}$
(B) $1+x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\frac{x^{4}}{8}+\frac{x^{5}}{15}+\frac{x^{6}}{48}$
(C) $1+2 x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\frac{x^{4}}{8}+\frac{x^{5}}{15}+\frac{x^{6}}{48}$
(D) $1+x+x^{2}+\frac{x^{3}}{3}+\frac{x^{4}}{8}+\frac{x^{5}}{15}+\frac{x^{6}}{48}$
27. For the ordinary differential equation $y^{\prime}=x \sqrt{y}$,
(A) there exists no general solution
(B) there exists exactly one particular solution
(C) $y=0$ is a singular solution
(D) $y=0$ is a trivial solution but not a singular solution
28. If $u(x, t)$ is a solution to the wave equation $u_{t t}=c^{2} u_{x x}$, (1) then which one of the following is not true ?
(A) A translate $u(x-d, t)$, where $d$ is a fixed real constant, may not be a solution to (1)
(B) Any derivative $u x(x, t)$ is also a solution to (1) if $u(x, t)$ is thrice differentiable
(C) Any dilation $u(b x, b t)$ is also a solution to (1) where $b$ is any fixed real constant
(D) For any twice differentiable function $R(x)$, the function. $u(x, t)=R(x-c t)$ is also a solution of (1)
29. If for all $x>0, e^{x}>x^{t}$, then
(A) $t>e$
(B) $t<e$
(C) $t=e$
(D) $t=e^{-1}$
30. If $\Gamma(x) \Gamma(1-x)=\frac{\pi}{\sin \pi x}$, then $\int_{0}^{\pi / 2}(\tan t)^{n} d t=$
(A) $\pi \sec \frac{\mathrm{n} \pi}{2}$
(B) $\frac{\pi}{2} \sec \frac{n \pi}{2}$
(C) $\frac{\pi}{2} \tan \frac{n \pi}{2}$
(D) $\pi \tan \frac{\mathrm{n} \pi}{2}$
31. If $X, Y, Z$ are pair-wise independent random variables then they
(A) are mutually independent
(B) are conditionally independent
(C) are degenerate
(D) need not be mutually independent
32. For detecting a disease, a test gives correct diagnosis with probability 0.99 . It is known that $1 \%$ of a population suffers from the disease. If a randomly selected individual from that population tests positive, then what is the probability that the selected individual actually has the disease ?
(A) 0.01
(B) 0.5
(C) 0.05
(D) 0.99
33. Which one of the following is not a characteristic function?
(A) $f(t)=e^{-|t|}$
(B) $f(t)=e^{-t^{2}}$
(C) $f(t)=e^{-t^{3}}$
(D) $f(t)=\frac{1}{1+t^{2}}$
34. Which one of the following is not true for an ergodic Markov Chain?
(A) It is irreducible
(B) It is periodic with period 2
(C) It is non-null
(D) It is recurrent
35. If $\{X(t), t \geq 0\}$ is a birth-death process with birth rate $\lambda_{n}=n \lambda$ and death rate $\mu_{\mathrm{n}}=\mathrm{n} \mu$ then the distribution of the Sojourn time of the process at state 1 is
(A) Exponential with mean $\mu^{-1}$
(B) Exponential with mean $\lambda^{-1}$
(C) Exponential with mean $(\lambda+\mu)^{-1}$
(D) Exponential with mean $\lambda(\lambda+\mu)^{-1}$
36. If $X$ follows an exponential distribution with unit mean then for $x>0, y>0$ :
(A) $P[X>x+y \mid X>x]=e^{-y}$
(B) $P[X>x+y \mid X>x]=e^{-x}$
(C) $P[X \leq x+y \mid X>x]=e^{-y}$
(D) $P[X \leq x+y \mid X>x]=e^{-x}$
37. Which one of the following statements is true for a standard Cauchy distribution?
(A) Distribution of the sample mean is again standard Cauchy
(B) All its moments are finite
(C) Its moment generating function is differentiable
(D) It is not a member of location - scale family
38. The least squares estimator coincides with the maximum likelihood estimator if the random sample is drawn from
(A) Exponential distribution
(B) Laplace distribution
(C) Cauchy distribution
(D) Normal distribution
39. Let $\left(X_{1}, X_{2}, \ldots, X_{n}\right)$ be a random sample from the density $f(x)=\frac{1}{2} e^{-|x-\mu|}$. Then the maximum likelihood estimation of $\mu$ is
(A) the sample mean
(B) the sample range
(C) the sample median
(D) the sample minimum
40. Let $\lambda\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ be the likelihood ratio test statistic for testing $\mathrm{H}_{0}: \mu=\mu_{0}$ against $H_{1}: \mu \neq \mu_{0}$ based on a random sample of size $n$ from a normal distribution with mean $\mu$ and unknown variance $\sigma^{2}$. Then asymptotic
distribution of $-2 \log \lambda\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ is
(A) Chi-square with one degrees of freedom
(B) Chi-square with 2 degrees of freedom
(C) Chi-square with n degrees of freedom
(D) t-distribution with $\mathrm{n}-1$ degrees of freedom
41. In a Gauss-Markov model $\left(Y, A \theta, \sigma^{2} I_{x}\right)$, which one of the following is not a condition for estimability of a linear parametric function $a^{\prime} \theta$ ?
(A) a belongs to the column space of $A$
(B) a belongs to the row space of $A$
(C) a belongs to the column space of $A^{\prime} A$
(D) a belongs to the estimation space
42. In a simple linear regression model $Y_{i}=\beta_{0}+\beta_{1} x_{i}+\epsilon_{i}, i=1,2, \ldots, n$, the variance of the least squares estimator of $\beta_{1}$ is
(A) $\frac{\sigma^{2}}{S_{x x}}$, when $S_{x x}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$
(B) $\sigma^{2}\left(1+\frac{\bar{x}^{2}}{S_{x x}}\right)$
(C) $\sigma^{2}\left(\frac{1}{n}+\frac{\bar{x}^{2}}{S_{x x}}\right)$
(D) $\sigma^{2}\left(\frac{\bar{x}^{2}}{S_{x x}}\right)$
43. The characteristic function of a Wishard distribution (denoted by $\mathrm{W}(\Sigma, \mathrm{n})$ ) is given by
(A) $\frac{\left|\Sigma^{-1}\right|}{\left|\Sigma^{-1}-2 i t\right|}$
(B)
$\left(\frac{\left|\Sigma^{-1}\right|}{\left|\Sigma^{-1}-2 \mathrm{it}\right|}\right)^{n}$
(C) $\left(\frac{|\Sigma|}{|\Sigma-2 i t|}\right)^{n / 2}$
(D) $\left(\frac{\left|\Sigma^{-1}\right|}{\left|\Sigma^{-1}-2 i t\right|}\right)^{n / 2}$
44. If the bivariate random vector $\left(X_{1}, X_{2}\right)$ has the dispersion matrix
$\sum=\left(\begin{array}{cc}1 & 4 \\ 4 & 100\end{array}\right)$, then the first principle component is
(A) $Y_{1}=-0.040 X_{1}+0.999 X_{2}$
(B) $Y_{1}=0.040 X_{1}+0.999 X_{2}{ }^{2}$
(C) $Y_{1}^{1}=0.999 X_{1}^{1}+0.040 \mathrm{X}_{2}^{2}$
(D) $Y_{1}^{1}=0.999 X_{1}^{1}-0.040 \mathrm{X}_{2}^{2}$
45. For a simple random sampling without replacement, the probability that a specified unit is included in the sample is
(A) $\frac{\mathrm{n}}{\mathrm{N}}$
(B) $\frac{1}{\mathrm{~N}}$
(C) $\frac{\mathrm{n}-1}{\mathrm{~N}-1}$
(D) $\frac{1}{\mathrm{~N}^{\mathrm{n}}}$
46. Which of the following need not be true for a BIBD ?
(A) It is proper
(B) It is variance balanced
(C) It is orthogonal
(D) It is binary
47. Given the principle block (1), $C, A B$, $A B C$, which of the following is confounded?
(A) AB
(B) AC
(C) $A B C$
(D) C
48. Hazard rate $h(t)$ of lognormal distribution is
(A) increasing in t
(B) decreasing in $t$
(C) constant in $t$
(D) non-monotonic in $t$
49. Which one of the following statements is NOT true ?
(A) Linear Programming Problem (LPP) is an optimization problem
(B) Every linear programming problem has a solution
(C) Solution space of an LPP is convex
(D) Dual of dual is primal
50. The steady state distribution of the number of customers in an $\mathrm{G}|\mathrm{M}| 1$ system at the arrival time points is
(A) Poisson
(B) Geometric
(C) Binomial
(D) Uniform

#  Space for Rough Work 

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